

# Standard Model Higgs inflation: CMB, Higgs mass and quantum cosmology

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We consider the inflation model generated by the Standard Model (SM) Higgs boson having a strong non-minimal curvature coupling. This model suggests the range of the Higgs mass  $135.6 \text{ GeV} \lesssim M_H \lesssim 184.5 \text{ GeV}$  entirely determined by the lower WMAP bound on the CMB spectral index. This result is based on the renormalization group analysis of quantum effects which make the SM phenomenology sensitive to the current cosmological data and thus suggest CMB measurements as a SM test complementary to the LHC program. We show naturalness of the gradient and curvature expansion in this model in a conventional perturbation theory range of SM. The origin of initial conditions for inflation within the quantum cosmology concept of the tunneling state of the Universe is also considered. In this way a complete cosmological scenario is obtained, which embraces the formation of initial conditions for the inflationary background in the form of a sharp probability peak in the distribution of the inflaton field and the ongoing generation of the CMB spectrum on this background.

## §1. Introduction

The goal of this work is an attempt of constructing a fundamental particle model accounting for an inflationary scenario and its observable CMB spectrum. An obvious rationale behind this is the anticipation that cosmological observations can comprise Standard Model (SM) tests complimentary to collider experiments – the line of thought essentially revived by the hope for the forthcoming discovery of the Higgs boson at LHC. While the relatively old work<sup>1)</sup> had suggested that due to quantum effects inflation depends not only on the inflaton-graviton sector of the system but rather is strongly effected by the GUT contents of the particle model, the series of papers<sup>2)–6)</sup> transcended this idea to the SM ground with the Higgs field playing the role of an inflaton. This has recovered interest in a once rather popular<sup>1),7)–9)</sup> model with the Lagrangian of the graviton-inflaton sector

$$L(g_{\mu\nu}, \Phi) = \frac{1}{2} (M_P^2 + \xi |\Phi|^2) R - \frac{1}{2} |\nabla \Phi|^2 - V(|\Phi|), \quad (1.1)$$

$$V(|\Phi|) = \frac{\lambda}{4} (|\Phi|^2 - v^2)^2, \quad |\Phi|^2 = \Phi^\dagger \Phi, \quad (1.2)$$

where  $\Phi$  is a scalar field whose expectation value plays the role of an inflaton and which has a strong non-minimal curvature coupling with  $\xi \gg 1$ . Here,  $M_P = m_P/\sqrt{8\pi} \approx 2.4 \times 10^{18} \text{ GeV}$  is a reduced Planck mass,  $\lambda$  is a quartic self-coupling of  $\Phi$ , and  $v$  is a symmetry breaking scale.

The motivation for this model was based on the observation<sup>7)</sup> that the problem

of an exceedingly small quartic coupling  $\lambda \sim 10^{-13}$ , dictated by the amplitude of primordial CMB perturbations,<sup>10)</sup> can be solved by using a non-minimally coupled inflaton with a large value of  $\xi$ . Later this model with the GUT-type sector of matter fields was used to generate initial conditions for inflation<sup>1)</sup> within the concept of the no-boundary<sup>11)</sup> and tunneling cosmological state.<sup>12)</sup> The quantum evolution with these initial data was considered in<sup>8),13)</sup> There it was shown that quantum effects are critically important for this scenario.

A similar theory was recently suggested<sup>2)</sup> but with the SM Higgs boson  $\Phi$  playing the role of an inflaton instead of the abstract GUT setup.<sup>1),8)</sup> This work advocated the consistency of the corresponding CMB data with WMAP observations at the tree-level approximation of the theory, which was extended in<sup>3)</sup> to the one-loop level. This has led to the lower bound on the Higgs mass  $M_H \gtrsim 230$  GeV, originating from the observational restrictions on the CMB spectral index.<sup>3)</sup> However, this conclusion which contradicts the widely accepted range  $115 \text{ GeV} \leq M_H \leq 180 \text{ GeV}$  did not take into account  $O(1)$  contributions due to renormalization group (RG) running, which qualitatively changes the situation. This was nearly simultaneously observed in<sup>4),5),14)</sup> where the RG improvement of the one-loop results of<sup>3)</sup> predicted the Higgs mass range almost coinciding with the conventional one.

Here we briefly report on the RG improvement of the one-loop results<sup>15)</sup> and suggest the CMB compatible range of the Higgs mass, both boundaries of this range being determined by the lower WMAP bound on the CMB spectral index,  $n_s \gtrsim 0.94$ , rather than by perturbation theory arguments. This makes the phenomenology of this gravitating SM essentially more sensitive to the cosmological bounds than in<sup>4)-6)</sup> and makes it testable by the CMB observations. Then we discuss the naturalness of the gradient and curvature expansion used for the derivation of the above result. Finally, we consider the origin of initial conditions for inflation within the concept of the tunneling state in quantum cosmology. In contrast to the conventional semi-classical approach to the minisuperspace Wheeler-DeWitt equation<sup>12)</sup> this state can be generated as a saddle-point contribution to the path integral for the statistical sum of the microcanonical ensemble in cosmology.<sup>16)-18)</sup> For massive quantum fields – the case of the dominant contribution of the heavy EW sector of SM – it can be consistently defined within the conventional UV renormalization and RG resummation,<sup>18)</sup> and its application yields the initial conditions for inflation in the form of a sharp probability peak for the distribution of the initial value of the inflaton field. In this way a complete cosmological scenario is obtained, which embraces the formation of initial conditions for the inflationary background and the ongoing generation of the CMB spectrum.

## §2. Quantum effects of the inflationary dynamics and CMB: the role of the anomalous scaling

The usual understanding of non-renormalizable theories is that renormalization of higher-dimensional operators does not effect the renormalizable sector of low-dimensional operators, because the former ones are suppressed by powers of a cutoff – the Planck mass  $M_P$ .<sup>19)</sup> Therefore, beta functions of the Standard Model sector

are not expected to be modified by gravitons. The situation with the non-minimal coupling is more subtle. Due to mixing of the Higgs scalar field with the longitudinal part of gravity in the kinetic term of the Lagrangian (1·1), an obvious suppression of pure graviton loops by the effective Planck mass,  $M_P^2 + \xi\varphi^2 \gg M_P^2$ , for large  $\xi$  proliferates to the sector of the Higgs field, so that certain parts of beta functions are strongly damped by large  $\xi$ .<sup>5)</sup> Therefore, a special combination of coupling constants  $\mathbf{A}$  which we call *anomalous scaling*<sup>1)</sup> becomes very small and brings down the CMB compatible Higgs mass bound. The importance of this quantity follows from the fact observed in<sup>1),3),8)</sup> that due to large  $\xi$ , quantum effects and their CMB manifestation are universally determined by  $\mathbf{A}$ . The nature of this quantity is as follows.

Let the model contain in addition to (1·1) also a set of scalar fields  $\chi$ , vector gauge bosons  $A_\mu$  and spinors  $\psi$ , which have an interaction with  $\Phi$  dictated by the local gauge invariance. If we denote by  $\varphi$  the inflaton – the only nonzero component of the mean value of  $\Phi$  in the cosmological state, then the quantum effective action of the system takes a generic form

$$S[g_{\mu\nu}, \varphi] = \int d^4x g^{1/2} \left( -V(\varphi) + U(\varphi) R(g_{\mu\nu}) - \frac{1}{2} G(\varphi) (\nabla\varphi)^2 + \dots \right), \quad (2·1)$$

where  $V(\varphi)$ ,  $U(\varphi)$  and  $G(\varphi)$  are the coefficients of the derivative expansion, and we disregard the contribution of higher-derivative operators negligible in the slow-roll approximation of the inflation theory. In this approximation the dominant quantum contribution to these coefficients comes from the heavy massive sector of the model. In particular, the masses of the physical particles and Goldstone modes  $m(\varphi)$ , generated by their quartic, gauge and Yukawa couplings with  $\varphi$ , give rise to the Coleman-Weinberg potential – the one-loop contribution to the effective potential  $V$  in (2·1). Since  $m(\varphi) \sim \varphi$ , for large  $\varphi$  this potential reads

$$V^{\text{1-loop}}(\varphi) = \sum_{\text{particles}} (\pm 1) \frac{m^4(\varphi)}{64\pi^2} \ln \frac{m^2(\varphi)}{\mu^2} = \frac{\lambda \mathbf{A}}{128\pi^2} \varphi^4 \ln \frac{\varphi^2}{\mu^2} + \dots \quad (2·2)$$

and, thus, determines the dimensionless coefficient  $\mathbf{A}$  – the anomalous scaling associated with the normalization scale  $\mu$  in (2·2). Moreover, for  $\xi \gg 1$  mainly this quantity and the dominant quantum correction to  $U(\varphi)$ ,<sup>15)</sup>

$$U^{\text{1-loop}}(\varphi) = \frac{3\xi\lambda}{32\pi^2} \varphi^2 \ln \frac{\varphi^2}{\mu^2} + \dots, \quad (2·3)$$

determine the quantum rolling force in the effective equation of the inflationary dynamics<sup>8),13)</sup> and yields the parameters of the CMB generated during inflation.<sup>3)</sup>

Inflation and its CMB are easy to analyze in the Einstein frame of fields  $\hat{g}_{\mu\nu}$ ,  $\hat{\varphi}$  in which the action  $\hat{S}[\hat{g}_{\mu\nu}, \hat{\varphi}] = S[g_{\mu\nu}, \varphi]$  has a minimal coupling, canonically normalized inflaton field and the new inflaton potential  $\hat{V} = M_P^4 V(\varphi)/4U^2(\varphi)$ <sup>\*)</sup>,

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<sup>\*)</sup> The Einstein and Jordan frames are related by the equations  $\hat{g}_{\mu\nu} = 2U(\varphi)g_{\mu\nu}/M_P^2$ ,  $(d\hat{\varphi}/d\varphi)^2 = M_P^2(GU + 3U'^2)/2U^2$ .

which reads at the inflation scale as

$$\hat{V} = \frac{M_P^4 V(\varphi)}{4U^2(\varphi)} \simeq \frac{\lambda M_P^4}{4\xi^2} \left( 1 - \frac{2M_P^2}{\xi\varphi^2} + \frac{\mathbf{A}_I}{16\pi^2} \ln \frac{\varphi}{\mu} \right). \quad (2.4)$$

Here the parameter  $\mathbf{A}_I$  represents the anomalous scaling (2.12) modified by the quantum correction to the non-minimal curvature coupling (2.3),

$$\mathbf{A}_I = \mathbf{A} - 12\lambda = \frac{3}{8\lambda} \left( 2g^4 + (g^2 + g'^2)^2 - 16g_t^4 \right) - 6\lambda. \quad (2.5)$$

This quantity – which we will call *inflationary anomalous scaling* – enters the expressions for the slow-roll parameters,  $\hat{\varepsilon} \equiv (M_P^2/2V^2)(d\hat{V}/d\hat{\varphi})^2$  and  $\hat{\eta} \equiv (M_P^2/\hat{V})d^2\hat{V}/d\hat{\varphi}^2$ , and ultimately determines all the inflation characteristics. In particular, smallness of  $\hat{\varepsilon}$  yields the range of the inflationary stage  $\varphi > \varphi_{\text{end}}$ , terminating at the value of  $\hat{\varepsilon}$ , which we chose to be  $\hat{\varepsilon}_{\text{end}} = 3/4$ . Then the inflaton value at the exit from inflation equals  $\varphi_{\text{end}} \simeq 2M_P/\sqrt{3\xi}$  under the natural assumption that perturbation expansion is applicable for  $\mathbf{A}_I/64\pi^2 \ll 1$ . The value of  $\varphi$  at the beginning of the inflation stage of duration  $N$  in units of the e-folding number then reads<sup>3)</sup>

$$\varphi^2 = \frac{4N}{3} \frac{M_P^2}{\xi} \frac{e^x - 1}{x}, \quad (2.6)$$

$$x \equiv \frac{N\mathbf{A}_I}{48\pi^2}, \quad (2.7)$$

where a special parameter  $x$  directly involves the anomalous scaling  $\mathbf{A}_I$ .

This relation determines the Fourier power spectrum for the scalar metric perturbation  $\zeta$ ,  $\Delta_\zeta^2(k) \equiv \langle k^3 \zeta_{\mathbf{k}}^2 \rangle = \hat{V}/24\pi^2 M_P^4 \hat{\varepsilon}$ , where the right-hand side is taken at the first horizon crossing,  $k = aH$ , relating the comoving perturbation wavelength  $k^{-1}$  to the e-folding number  $N$ ,

$$\Delta_\zeta^2(k) = \frac{N^2}{72\pi^2} \frac{\lambda}{\xi^2} \left( \frac{e^x - 1}{x e^x} \right)^2. \quad (2.8)$$

The CMB spectral index  $n_s \equiv 1 + d \ln \Delta_\zeta^2 / d \ln k = 1 - 6\hat{\varepsilon} + 2\hat{\eta}$  and the tensor to scalar ratio  $r = 16\hat{\varepsilon}$  correspondingly read as<sup>\*)</sup>

$$n_s = 1 - \frac{2}{N} \frac{x}{e^x - 1}, \quad (2.9)$$

$$r = \frac{12}{N^2} \left( \frac{x e^x}{e^x - 1} \right)^2. \quad (2.10)$$

Therefore, with the spectral index constraint  $0.94 < n_s(k_0) < 0.99$  (the combined WMAP+BAO+SN data at the pivot point  $k_0 = 0.002 \text{ Mpc}^{-1}$  corresponding to  $N \simeq 60^{21})$  these relations immediately give the range of anomalous scaling  $-12 < \mathbf{A}_I < 14$ .<sup>3)</sup>

<sup>\*)</sup> Note that for  $|x| \ll 1$  these predictions exactly coincide with those<sup>25)</sup> of the  $f(R) = M_P^2(R + R^2/6M^2)/2$  inflationary model<sup>26)</sup> with the scalar particle (scalarm) mass  $M = M_P \sqrt{\lambda}/\sqrt{3\xi}$ .

On the other hand, in the Standard Model  $\mathbf{A}$  is expressed in terms of the masses of the heaviest particles –  $W^\pm$  boson,  $Z$  boson and top quark,

$$m_W^2 = \frac{1}{4} g^2 \varphi^2, \quad m_Z^2 = \frac{1}{4} (g^2 + g'^2) \varphi^2, \quad m_t^2 = \frac{1}{2} y_t^2 \varphi^2, \quad (2.11)$$

and the mass of three Goldstone modes  $m_G^2 = V'(\varphi)/\varphi = \lambda(\varphi^2 - v^2) \simeq \lambda\varphi^2$ . Here,  $g$  and  $g'$  are the  $SU(2) \times U(1)$  gauge couplings,  $g_s$  is the  $SU(3)$  strong coupling, and  $y_t$  is the Yukawa coupling for the top quark. At the inflation stage the Goldstone mass  $m_G^2$  is non-vanishing in contrast to its zero on-shell value in the electroweak vacuum.<sup>22)</sup> Therefore, Eq. (2.2) gives the expression

$$\mathbf{A} = \frac{3}{8\lambda} \left( 2g^4 + (g^2 + g'^2)^2 - 16y_t^4 \right) + 6\lambda. \quad (2.12)$$

In the conventional range of the Higgs mass  $115 \text{ GeV} \leq M_H \leq 180 \text{ GeV}$ <sup>23)</sup> this quantity at the electroweak scale belongs to the range  $-48 < \mathbf{A} < -20$  which strongly contradicts the CMB range given above.

However, the RG running of coupling constants is strong enough and drives  $\mathbf{A}$  to the CMB compatible range at the inflation scale. Below we show that the formalism of<sup>3)</sup> stays applicable but with the electroweak  $\mathbf{A}$  replaced by the running  $\mathbf{A}(t)$ , where  $t = \ln(\varphi/\mu)$  is the running scale of the renormalization group (RG) improvement of the effective potential.<sup>24)</sup>

### §3. Renormalization group improvement

According to the Coleman-Weinberg technique<sup>24)</sup> the one-loop RG improved effective action has the form (2.1) with

$$V(\varphi) = \frac{\lambda(t)}{4} Z^4(t) \varphi^4, \quad (3.1)$$

$$U(\varphi) = \frac{1}{2} \left( M_P^2 + \xi(t) Z^2(t) \varphi^2 \right), \quad (3.2)$$

$$G(\varphi) = Z^2(t). \quad (3.3)$$

Here the running scale  $t = \ln(\varphi/M_t)$  is normalized at the top quark mass  $\mu = M_t$  (we denote physical (pole) masses by capital letters in contrast to running masses (2.11) above). The running couplings  $\lambda(t)$ ,  $\xi(t)$  and the field renormalization  $Z(t)$  incorporate summation of powers of logarithms and belong to the solution of the RG equations

$$\frac{dg_i}{dt} = \beta_{g_i}, \quad \frac{dZ}{dt} = \gamma Z \quad (3.4)$$

for the full set of coupling constants  $g_i = (\lambda, \xi, g, g', g_s, y_t)$  in the “heavy” sector of the model with the corresponding beta functions  $\beta_{g_i}$  and the anomalous dimension  $\gamma$  of the Higgs field.

An important subtlety with these  $\beta$  functions is the effect of non-minimal curvature coupling of the Higgs field. For large  $\xi$  the kinetic term of the tree-level action

has a strong mixing between the graviton  $h_{\mu\nu}$  and the quantum part of the Higgs field  $\sigma$  on the background  $\varphi$ . Symbolically it has the structure

$$(M_P^2 + \xi^2 \varphi^2) h \nabla \nabla h + \xi \varphi \sigma \nabla \nabla h + \sigma \square \sigma,$$

which yields a propagator whose elements are suppressed by a small  $1/\xi$ -factor in all blocks of the  $2 \times 2$  graviton-Higgs sector. For large  $\varphi \gg M_P/\sqrt{\xi}$ , the suppression of pure graviton loops is, of course, obvious because of the effective Planck mass squared essentially exceeding the Einstein one,  $M_P^2 + \xi \varphi^2 \gg M_P^2$ . Due to mixing, this suppression proliferates to the full graviton-Higgs sector of the theory and gives the Higgs propagator  $s(\varphi)/(\square - m_H^2)$  weighted by the suppression factor  $s(\varphi)$

$$s(\varphi) = \frac{M_P^2 + \xi \varphi^2}{M_P^2 + (6\xi + 1)\xi \varphi^2}. \quad (3.5)$$

This mechanism<sup>8),13),27)</sup> modifies the beta functions of the SM sector<sup>5)</sup> at high energy scales because the factor  $s(\varphi)$ , which is close to one at the EW scale  $v \ll M_P/\xi$ , is very small for  $\varphi \gg M_P/\sqrt{\xi}$ ,  $s \simeq 1/6\xi$ . Such a modification, in fact, justifies the extension beyond the scale  $M_P/\xi$  interpreted in<sup>28),29)</sup> as a natural validity cutoff of the theory\*).

There is an important subtlety with the modification of beta functions which was disregarded in<sup>5)</sup> (and in the first version of<sup>15)</sup>). Goldstone modes, in contrast to the Higgs particle, do not have a kinetic term mixing with gravitons.<sup>6)</sup> Therefore, their contribution is not suppressed by the  $s$ -factor of the above type. Separation of Goldstone contributions from the Higgs ones leads to the following modification of the one-loop beta functions, which is essentially different from that of<sup>5)</sup> (cf. also<sup>31)</sup>)

$$\beta_\lambda = \frac{\lambda}{16\pi^2} (18s^2\lambda + \mathbf{A}(t)) - 4\gamma\lambda, \quad (3.6)$$

$$\beta_\xi = \frac{6\xi}{16\pi^2} (1 + s^2)\lambda - 2\gamma\xi, \quad (3.7)$$

$$\beta_{y_t} = \frac{y_t}{16\pi^2} \left( -\frac{2}{3}g'^2 - 8g_s^2 + \left(1 + \frac{s}{2}\right) y_t^2 \right) - \gamma y_t, \quad (3.8)$$

$$\beta_g = -\frac{39-s}{12} \frac{g^3}{16\pi^2}, \quad (3.9)$$

$$\beta_{g'} = \frac{81+s}{12} \frac{g'^3}{16\pi^2}, \quad (3.10)$$

$$\beta_{g_s} = -\frac{7g_s^3}{16\pi^2}. \quad (3.11)$$

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<sup>\*)</sup> The smallness of this cutoff could be interpreted as inefficiency of the RG analysis beyond the range of trustability of the model. However, the cutoff  $M_P/\xi \ll M_P$  of<sup>28),29)</sup> applies to energies (momenta) of scattering processes in flat spacetime with a small EW value of  $\varphi$ . For the inflation stage on the background of a large  $\varphi$  this cutoff gets modified due to the increase in the effective Planck mass  $M_P^2 + \xi \varphi^2 \gg M_P^2$  (and the associated decrease of the  $s$ -factor (3.5) – resummation of terms treated otherwise as perturbations in<sup>28)</sup>). Thus the magnitude of the Higgs field at inflation is not really indicative of the violation of the physical cutoff bound (see discussion in Sects. 5 and 6 below).

Here the anomalous dimension of the Higgs field  $\gamma$  is given by a standard expression in the Landau gauge

$$\gamma = \frac{1}{16\pi^2} \left( \frac{9g^2}{4} + \frac{3g'^2}{4} - 3y_t^2 \right), \quad (3.12)$$

the anomalous scaling  $\mathbf{A}(t)$  is defined by (2.12) and we retained only the leading terms in  $\xi \gg 1$ . It will be important in what follows that this anomalous scaling contains the Goldstone contribution  $6\lambda$ , so that the full  $\beta_\lambda$  in (3.6) has a  $\lambda^2$ -term unsuppressed by  $s(\varphi)$  at large scale  $t = \ln(\varphi/\mu)$ .

The inflationary stage in units of Higgs field e-foldings is very short, which allows one to use the approximation linear in  $\Delta t \equiv t - t_{\text{end}} = \ln(\varphi/\varphi_{\text{end}})$ , where the initial data point is chosen at the end of inflation  $t_{\text{end}}$ . Therefore, for beta functions (3.6) and (3.7) with  $s \ll 1$  we have

$$\lambda(t) = \lambda_{\text{end}} \left( 1 - 4\gamma_{\text{end}} \Delta t + \frac{\mathbf{A}(t_{\text{end}})}{16\pi^2} \Delta t \right), \quad (3.13)$$

$$\xi(t) = \xi_{\text{end}} \left( 1 - 2\gamma_{\text{end}} \Delta t + \frac{6\lambda_{\text{end}}}{16\pi^2} \Delta t \right), \quad (3.14)$$

where  $\lambda_{\text{end}}$ ,  $\gamma_{\text{end}}$ ,  $\xi_{\text{end}}$  are determined at  $t_{\text{end}}$  and  $\mathbf{A}_{\text{end}} = \mathbf{A}(t_{\text{end}})$  is also a particular value of the running anomalous scaling (2.12) at the end of inflation.

On the other hand, the RG improvement of the effective action (3.1)-(3.3) implies that this action coincides with the tree-level action for a new field  $\phi = Z(t)\varphi$  with running couplings as functions of  $t = \ln(\varphi/\mu)$  (the running of  $Z(t)$  is slow and affects only the multi-loop RG improvement). Then, in view of (3.1)-(3.2) the RG improved potential takes at the inflation stage the form of the one-loop potential (2.4) for the field  $\phi$  with a particular choice of the normalization point  $\mu = \phi_{\text{end}}$  and all the couplings replaced by their running values taken at  $t_{\text{end}}$ . Therefore, the formalism of<sup>3)</sup> can be directly applied to find the CMB parameters of the model, which now turn out to be determined by the running anomalous scaling  $\mathbf{A}_I(t)$  taken at  $t_{\text{end}}$ .

In contrast to the inflationary stage, the post-inflationary running is very large and requires numerical simulation. We fix the  $t = 0$  initial conditions for the RG equations (3.4) at the top quark scale  $M_t = 171$  GeV. For the constants  $g, g'$  and  $g_s$ , they read<sup>23)</sup>

$$g^2(0) = 0.4202, \quad g'^2(0) = 0.1291, \quad g_s^2(0) = 1.3460, \quad (3.15)$$

where  $g^2(0)$  and  $g'^2(0)$  are obtained by a simple one-loop RG flow from the conventional values of  $\alpha(M_Z) \equiv g^2/4\pi = 0.0338$ ,  $\alpha'(M_Z) \equiv g'^2/4\pi = 0.0102$  at  $M_Z$ -scale, and the value  $g_s^2(0)$  at  $M_t$  is generated by the numerical program of.<sup>32)</sup> The analytical algorithm of transition between different scales for  $g_s^2$  was presented in.<sup>33)</sup> For the Higgs self-interaction constant  $\lambda$  and for the Yukawa top quark interaction constant  $y_t$  the initial conditions are determined by the pole mass matching scheme originally developed in<sup>34)</sup> and presented in the Appendix of.<sup>35)</sup>

The initial condition  $\xi(0)$  follows from the CMB normalization (2.8),  $\Delta_\zeta^2 \simeq 2.5 \times 10^{-9}$  at the pivot point  $k_0 = 0.002 \text{ Mpc}^{-1}$ , which we choose to correspond to

$N \simeq 60$ .<sup>21)</sup> This yields the following estimate on the ratio of coupling constants

$$\frac{1}{Z_{\text{in}}^2} \frac{\lambda_{\text{in}}}{\xi_{\text{in}}^2} \simeq 0.5 \times 10^{-9} \left( \frac{x_{\text{in}} \exp x_{\text{in}}}{\exp x_{\text{in}} - 1} \right)^2 \quad (3.16)$$

at the moment of the first horizon crossing for  $N = 60$  which we call the “beginning” of inflation and label by  $t_{\text{in}} = \ln(\varphi_{\text{in}}/M_t)$  with  $\varphi_{\text{in}}$  defined by (2.6). Thus, the RG equations (3.4) for six couplings  $(g, g', g_s, y_t, \lambda, \xi)$  with five initial conditions and the final condition for  $\xi$  uniquely determine the needed RG flow.

The RG flow covers also the inflationary stage from the chronological end of inflation  $t_{\text{end}}$  to  $t_{\text{in}}$ . At the end of inflation we choose the value of the slow roll parameter  $\hat{\varepsilon} = 3/4$ , and  $\varphi_{\text{end}} \equiv M_t e^{t_{\text{end}}} \simeq M_P \sqrt{4/3} \xi_{\text{end}}$ . Thus, the duration of inflation in units of inflaton field e-foldings  $t_{\text{in}} - t_{\text{end}} = \ln(\varphi_{\text{in}}/\varphi_{\text{end}}) \simeq \ln N/2 \sim 2$  is very short relative to the post-inflationary evolution  $t_{\text{end}} \sim 35$ . The approximation linear in logs implies the bound  $|\mathbf{A}_I(t_{\text{end}})| \Delta t / 16\pi^2 \ll 1$ , which in view of  $\Delta t < t_{\text{in}} - t_{\text{end}} \simeq \ln N/2$  holds for  $|\mathbf{A}_I(t_{\text{end}})| / 16\pi^2 \ll 0.5$ .

#### §4. CMB compatible bounds on the Higgs mass

The running of  $\mathbf{A}(t)$  strongly depends on the behavior of  $\lambda(t)$ . For small Higgs masses the usual RG flow in SM leads to an instability of the EW vacuum caused by negative values of  $\lambda(t)$  in a certain range of  $t$ .<sup>35),36)</sup> The same happens in the presence of non-minimal curvature coupling. The numerical solution for  $\lambda(t)$  is shown

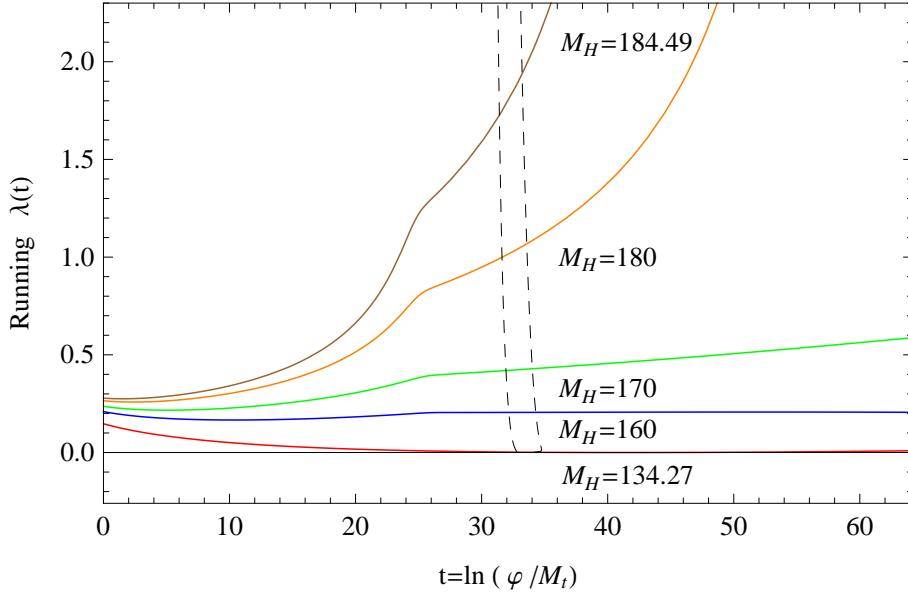


Fig. 1. Running  $\lambda(t)$  for five values of the Higgs mass above the instability threshold. Dashed curves mark the boundaries of the inflation domain  $t_{\text{end}} \leq t \leq t_{\text{in}}$ .

in Fig.1 for five values of the Higgs mass and the value of top quark mass  $M_t = 171$

GeV. The lowest one corresponds to the boundary of the instability window,

$$M_H^{\text{inst}} \simeq 134.27 \text{ GeV}, \quad (4.1)$$

for which  $\lambda(t)$  bounces back to positive values after vanishing at  $t_{\text{inst}} \sim 41.6$  or  $\varphi_{\text{inst}} \sim 80M_P$ . It turns out that the corresponding  $\xi(t)$  is nearly constant and is about 5000 (see below), so that the factor (3.5) at  $t_{\text{inst}}$  is very small  $s \simeq 1/6\xi \sim 0.00005$ . Thus the situation is different from the usual Standard Model with  $s = 1$ , and numerically the critical value turns out to be higher than the known SM stability bound  $\sim 125$  GeV.<sup>35)</sup>

Fig.1 shows that near the instability threshold  $M_H = M_H^{\text{inst}}$  the running coupling  $\lambda(t)$  stays very small for all scales  $t$  relevant to the observable CMB. This follows from the fact that the positive running of  $\lambda(t)$  caused by the term  $(18s^2 + 6)\lambda^2$  in  $\beta_\lambda$ , (3.6), is much slower for  $s \ll 1$  than that of the usual SM driven by the term  $24\lambda^2$ .

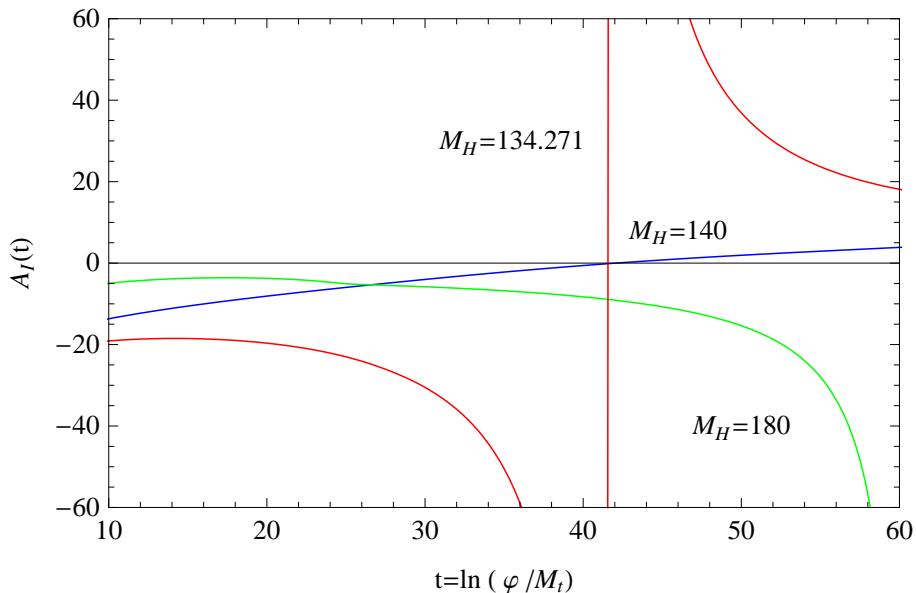


Fig. 2. Running anomalous scaling for the critical Higgs mass (the red curve with a vertical segment at the singularity with  $t_{\text{inst}} \sim 41.6$ ) and for two masses in the stability domain (blue and green curves).

For all Higgs masses in the range  $M_H^{\text{inst}} = 134.27$  GeV  $< M_H < 185$  GeV the inflation range  $t_{\text{end}} < t < t_{\text{in}}$  is always below  $t_{\text{inst}} = 41.6$ , so that from Fig. 2  $\mathbf{A}_I(t)$  is always negative during inflation. Its running explains the main difference from the results of one-loop calculations.<sup>3)</sup>  $\mathbf{A}_I(t)$  runs from big negative values  $\mathbf{A}_I(0) < -20$  at the electroweak scale to small also negative values at the inflation scale below  $t_{\text{inst}}$ . This makes the CMB data compatible with the generally accepted Higgs mass range. Indeed, the knowledge of the RG flow immediately allows one to obtain  $\mathbf{A}_I(t_{\text{end}})$  and  $x_{\text{end}}$  and thus find the parameters of the CMB power spectrum (2.9)-(2.10) as functions of  $M_H$ . The parameter of primary interest – spectral index

– is given by Eq. (2.9) with  $x = x_{\text{end}} \equiv N\mathbf{A}_I(t_{\text{end}})/48\pi^2$  and depicted in Fig.3. Even for low values of Higgs mass above the stability bound,  $n_s$  falls into the range admissible by the CMB constraint existing now at the  $2\sigma$  confidence level (based on the combined WMAP+BAO+SN data<sup>21)</sup>)  $0.94 < n_s(k_0) < 0.99$ .

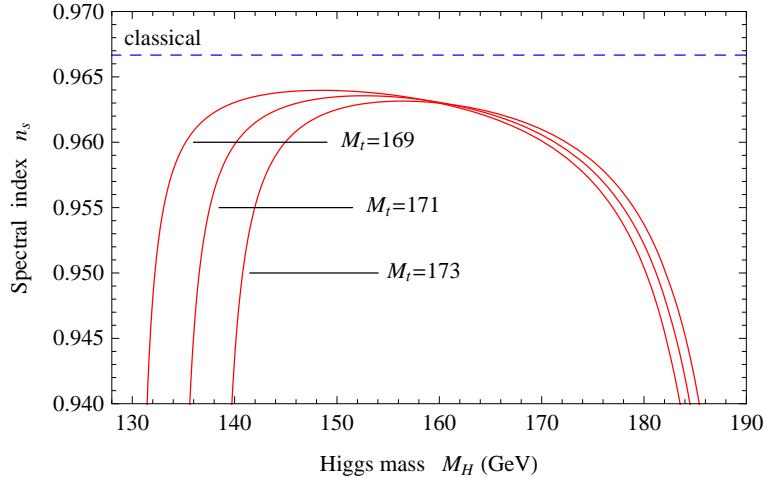


Fig. 3. The spectral index  $n_s$  as a function of the Higgs mass  $M_H$  for three values of the top quark mass.

The spectral index drops below 0.94 only for large  $x_{\text{end}} < 0$  or large negative  $\mathbf{A}_I(t_{\text{end}})$ , which happens only when  $M_H$  either approaches the instability bound or exceeds 180 GeV at the decreasing branch of the  $n_s$  graph. Thus, we get the lower and upper bounds on the Higgs mass, which both follow from the lower bound of the CMB data. Numerical analysis for the corresponding  $x_{\text{end}} \simeq -1.4$  gives for  $M_t = 171$  GeV the range of CMB compatible Higgs mass

$$135.62 \text{ GeV} \lesssim M_H \lesssim 184.49 \text{ GeV}. \quad (4.2)$$

Both bounds belong to the nonlinear domain of the equation (2.9) with  $x_{\text{end}} \simeq -1.4$ , but the quantity  $|\mathbf{A}_I(t_{\text{end}})|/16\pi^2 = 0.07 \ll 0.5$  satisfies the restriction mentioned above, and their calculation is still in the domain of our linear in logs approximation.

The upper bound on  $n_s$  does not generate restrictions on  $M_H$ . The lower CMB bound in (4.2) is slightly higher than the instability bound  $M_H^{\text{inst}} = 134.27$  GeV. In turn, this bound depends on the initial data for weak and strong couplings and on the top quark mass  $M_t$  which is known with less precision. The above bounds were obtained for  $M_t = 171$  GeV. Results for the neighboring values  $M_t = 171 \pm 2$  GeV are presented in Fig. 3 to show the pattern of their dependence on  $M_t$ .

Finally let us focus on the running of  $\xi(t)$ . It is very slow for low values of the Higgs mass near the instability threshold. Of course, this follows from the smallness of the running  $\lambda(t)$  in this domain. Another property of the  $\xi$ -behavior is that the normalization of the power spectrum leads to a value  $\xi \sim 5000$  for small Higgs masses, which is smaller than the old estimate<sup>1)-3), 7), 8), 13)</sup>  $\sim 10^4$ . This is caused by a decrease of  $\lambda(t)$  which at  $t_{\text{in}}$  becomes much smaller than  $\lambda(0)$ .<sup>5)</sup>

## §5. Gradient and curvature expansion cutoff and naturalness

The expression (2.1) is a truncation of the curvature and derivative expansion of the full effective action. It was repeatedly claimed that with large  $\xi$  the weak field version of this expansion on flat (and empty) space background has a cutoff  $4\pi M_P/\xi$ .<sup>28),29)</sup> This scale is essentially lower than the Higgs field during inflation  $\varphi \sim M_P/\sqrt{\xi}$  and, therefore, seems to invalidate predictions based on (2.1) without unnatural suppression of higher-dimensional operators.<sup>37)</sup> The attempt to improve the situation by transition to the Einstein frame<sup>38),39)</sup> was claimed to fail<sup>40),41)</sup> in view of a multiplet nature of the Higgs field involving Nambu-Goldstone modes.

Here we show that these objections against naturalness are not quite conclusive. First, as mentioned above, a big value of  $\varphi$  during inflation is not really indicative of a large physical scale of the problem. In contrast to curvature and energy density the inflaton itself is not a physical observable, but rather a configuration space coordinate of the model. Secondly, we show now that the inflation scale actually lies below the gradient expansion cutoff, and this justifies naturalness of the obtained results. No transition to another conformal frame is needed for that, but rather the resummation accounting for transition to large  $\varphi$  background.

Indeed, the main peculiarity of the model (1.1) is that in the background field method with small derivatives the role of the effective Planck mass is played by  $(M_P^2 + \xi\varphi^2)^{1/2}$ . The power-counting method<sup>28)</sup> underlying the derivation of the cutoff  $4\pi M_P/\xi$  also applies here but with the Planck mass  $M_P$  replaced by the effective one,  $M_P \rightarrow (M_P^2 + \xi\varphi^2)^{1/2} > \sqrt{\xi}\varphi$ . The resulting cutoff is thus bounded from below by

$$\Lambda(\varphi) = \frac{4\pi\varphi}{\sqrt{\xi}}, \quad (5.1)$$

and this bound can be used as a *running* cutoff of the gradient and curvature expansion. The origin of this cutoff can be demonstrated in the one-loop approximation. When calculated in the *Jordan* frame, the one-loop divergences quadratic in the curvature  $R$  have a strongest in  $\xi$  contribution (this can be easily deduced from the Appendix of<sup>15)</sup>)

$$\xi^2 \frac{R^2}{16\pi^2}. \quad (5.2)$$

As compared to the tree-level part linear in the curvature  $\sim (M_P^2 + \xi\varphi^2)R$ , the one loop  $R^2$ -term turns out to be suppressed by the above cutoff factor  $16\pi^2(M_P^2 + \xi\varphi^2)/\xi^2 \simeq \Lambda^2$ .

The on-shell curvature estimate at the inflation stage reads  $R \sim V/U \sim \lambda\varphi^2/\xi$ , so that the resulting curvature expansion runs in powers of

$$\frac{R}{\Lambda^2} \sim \frac{\lambda}{16\pi^2} \quad (5.3)$$

and remains efficient in the usual perturbation theory range of SM,  $\lambda/16\pi^2 \ll 1$ . This works perfectly well in our Higgs inflation model, because in the full CMB compatible range of the Higgs mass  $\lambda < 2$  (see Fig.1).

From the viewpoint of the gradient expansion for  $\varphi$  this cutoff is even more efficient. Indeed, the inflaton field gradient can be expressed in terms of the inflaton potential  $\hat{V}$  and the inflation smallness parameter  $\hat{\varepsilon}$  taken in the Einstein frame,  $\dot{\varphi} \simeq (\varphi^2/M_P^2)(\xi\hat{\varepsilon}\hat{V}/18)^{1/2}$ . With  $\hat{V} \simeq \lambda M_P^4/4\xi^2$  this immediately yields the gradient expansion in powers of

$$\frac{\partial}{\Lambda} \sim \frac{1}{\Lambda} \frac{\dot{\varphi}}{\varphi} \simeq \frac{\sqrt{\lambda}}{48\pi} \sqrt{2\hat{\varepsilon}}, \quad (5.4)$$

which is even better than (5.3) by the factor ranging from  $1/N$  at the beginning of inflation to  $O(1)$  at the end of it.

Eqs. (5.3) and (5.4) justify the effective action truncation in (2.1) in the inflationary domain. Thus only multi-loop corrections to the coefficient functions  $V(\varphi)$ ,  $U(\varphi)$  and  $G(\varphi)$  might stay beyond control in the form of higher-dimensional operators  $(\varphi/\Lambda)^n$  and violate the flatness of the effective potential necessary for inflation. However, in view of the form of the running cutoff (5.1) they might be large, but do not affect the shape of these coefficient functions because of the field independence of the ratio  $\varphi/\Lambda$ . Only the logarithmic running of couplings in (3.1)-(3.3) controlled by RG dominates the quantum input in the inflationary dynamics and its CMB spectra\*).

In this connection the results of the recent work<sup>39)</sup> might be helpful. It claims that due to asymptotic shift symmetry of the model (or asymptotic scale invariance in the Jordan frame) the field-dependent cutoff at the inflation scale is much higher than (5.1) and is given by  $\Lambda_E \sim \sqrt{\xi}\varphi$ , which strongly supports naturalness of Higgs inflation along with its consistency at the reheating and Big Bang stages. The difference from (5.1) can be explained by the fact that, in contrast to our Jordan frame calculations, the quantum corrections in<sup>39)</sup> were analyzed in the Einstein frame. In this frame, in particular, the strongest curvature squared counterterm is  $O(1)R^2/16\pi^2$  rather than (5.2) (see<sup>27)</sup>). Of course, modulo the conformal anomaly contribution, which only effects the log arguments and cannot be responsible for the onshell discrepancy in these counterterms, the physical results should be equivalent in both parameterizations.<sup>15)</sup> This can be an indication of intrinsic cancellations which are not manifest in the Jordan frame and which could effectively raise the cutoff from its naive value (5.1) to  $\Lambda_E$ . This would also justify suppression of higher dimensional operators in the coefficient functions  $V, U, G$  mentioned above. However, verification of frame equivalence of the physical results should be based on gauge and parametrization invariant definition of CMB observables which is currently under study.

## §6. Quantum cosmology origin of Higgs inflation

Now we want to show that, in addition to the good agreement of the CMB spectrum with the observational data, this model can also describe the mechanism of generating the cosmological *background* itself upon which these perturbations ex-

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<sup>\*)</sup> Like the logarithmic term of (2.4) which dominates over the nearly flat classical part of the inflaton potential and qualitatively modifies tree-level predictions of the theory.<sup>3)</sup>

ist. This mechanism consists in the formation of the initial conditions for inflation in the form of a sharp probability peak in the distribution function of the initial value of the inflaton field. Such a distribution naturally arises in quantum cosmology within two known suggestions for the wavefunction of the Universe – the no-boundary Hartle-Hawking prescription<sup>11)</sup> and the prescription of the tunneling cosmological wavefunction.<sup>12)</sup> Though originally these prescriptions did not have a good justification at the level of a consistent operator quantization in the physical spacetime with the Lorentzian signature\*), recently they were both derived as saddle-point contributions to the statistical sum of the microcanonical ensemble in cosmology (described by the density matrix which is just the projector onto the space of solutions of the Wheeler-DeWitt equations).<sup>16)–18)</sup>

As it was recently shown,<sup>18),42)</sup> specifically the tunneling state gets realized for heavy massive quantum fields\*\*) – exactly the case of the Higgs inflation with quantum corrections generated by a heavy EW sector of the Standard Model. Within the relevant inverse mass expansion (which is just the slow-roll approximation in cosmology or the expansion in terms of higher-dimensional operators of the effective field theory) the distribution function of the inflaton field turns out to be given by the exponentiated *Euclidean effective* action,  $\rho_{\text{tunnel}}(\varphi) = \exp(+S_E(\varphi))$  \*\*\*). This action – the Euclidean version of (2.1) – is supposed to be calculated on the (quasi)-de Sitter instanton of a spherical topology  $S^4$  of the size determined by the magnitude of the inflaton field and its effective inflaton potential. For our model of Higgs inflation with a non-minimal curvature coupling this is the Einstein frame potential (2.4), and the tunneling distribution function

$$\rho_{\text{tunnel}}(\varphi) = \exp\left(-\frac{24\pi^2 M_P^4}{\hat{V}(\varphi)}\right) \quad (6.1)$$

describes the ensemble of (quasi)de Sitter universes starting to expand with various values of the effective cosmological constant  $\hat{\Lambda} = \hat{V}(\varphi)/M_P^2$ . So the question is whether this distribution can have a sharp probability peak at some appropriate value of the inflaton field  $\varphi_0$  with which the Universe as a whole starts its evolution. The shape and the magnitude of  $\hat{V}$  depicted in Fig.4 for several values of the Higgs mass clearly indicates the existence of such a peak.

Indeed, the negative of the inverse potential damps to zero after exponentiation the probability of those values of  $\varphi$  at which  $\hat{V}(\varphi) = 0$  and, vice versa, enhances the probability at the positive maxima of the potential. The pattern of this behavior with growing Higgs mass  $M_H$  is as follows.

We begin with the EW vacuum instability threshold<sup>35),36)</sup> which exists in this gravitating SM at  $M_H^{\text{inst}} \approx 134.27$  GeV<sup>15)</sup> and which is slightly lower than the CMB

\*) The no-boundary state was put forward as a formal Euclidean quantum gravity path integral,<sup>11)</sup> whereas the tunneling prescription existed merely as a semiclassical solution of the minisuperspace Wheeler-DeWitt equation.<sup>12)</sup>

\*\*) In contrast to the case of massless conformal fields which give rise to the thermal version of the no-boundary state.<sup>42)</sup>

\*\*\*) Note the plus sign in the exponential, which is different from the minus sign corresponding to the no-boundary case.<sup>18),42)</sup>

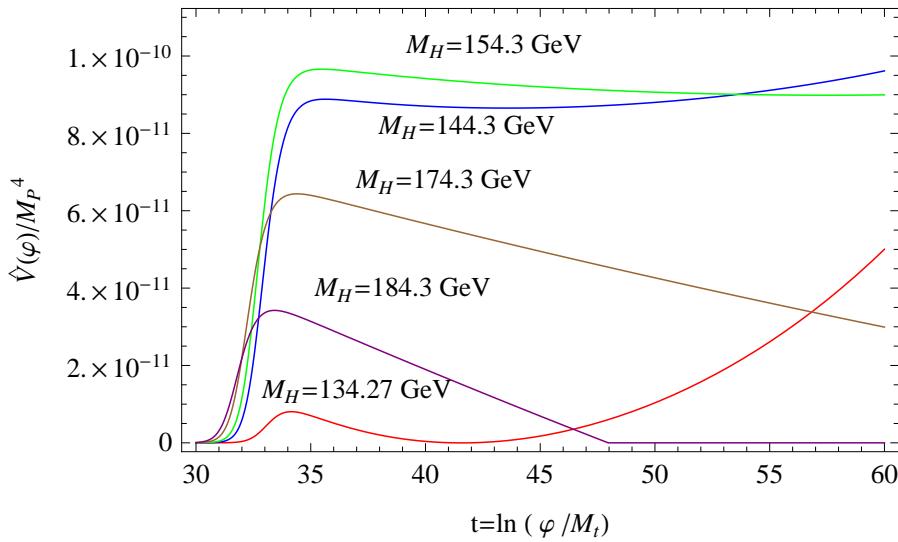


Fig. 4. The succession of effective potential graphs above the EW vacuum instability threshold  $M_H^{\text{inst}} = 134.27$  GeV up to  $M_H = 184.3$  GeV showing the occurrence of a metastable vacuum followed for high  $M_H$  by the formation of a negative slope branch. Local peaks of  $\hat{V}$  situated at  $t = 34 \div 35$  grow with  $M_H$  for  $M_H \lesssim 160$  GeV and start decreasing for larger  $M_H$ .

compatible range of the Higgs mass ( $M_H^{\text{inst}}$  is chosen in Fig. 5 and for the lowest curve in Fig. 4). The potential  $\hat{V}(\varphi)$  drops to zero at  $t_{\text{inst}} \simeq 41.6$ ,  $\varphi_{\text{inst}} \sim 80M_P$ , and forms a false vacuum<sup>15)</sup> separated from the EW vacuum by a large peak at  $t \simeq 34$ . Correspondingly, the probability of creation of the Universe with the initial value of the inflaton field at the EW scale  $\varphi = v$  and at the instability scale  $\varphi_{\text{inst}}$  is damped to zero, while the most probable value belongs to this peak. The inflationary stage of the formation of the pivotal  $N = 60$  CMB perturbation (from the moment  $t_{\text{in}}$  of the first horizon crossing until the end of inflation  $t_{\text{end}}$ ), which is marked by dashed lines in Fig. 5, lies to the left of this peak. This conforms to the requirement of the chronological succession of the initial conditions for inflation and the formation of the CMB spectra.

The above case is, however, below the CMB-compatible range of  $M_H$  and was presented here only for illustrative purposes. An important situation occurs at higher Higgs masses from the lower CMB bound on  $M_H \simeq 135.6$  GeV until about 160 GeV. Here we get a family of a metastable vacua with  $\hat{V} > 0$ . An example is the plot with  $M_H = 144.3$  GeV in Fig. 4. Despite the shallowness of this vacuum the small maximum of  $\hat{V}$  at  $t \simeq 35$  generates via (6.1) a sharp probability peak for the initial inflaton field. This follows from an extremely small value of  $\hat{V}/M_P^4 \sim 10^{-11}$ , the reciprocal of which generates a rapidly changing exponential of (6.1).

For even larger  $M_H$  these metastable vacua get replaced by a negative slope of the potential which interminably decreases to zero at large  $t$  (at least within the perturbation theory range of the model), see Fig. 4. Therefore, for large  $M_H$  close to the upper CMB bound 185 GeV, the probability peak of (6.1) gets separated from

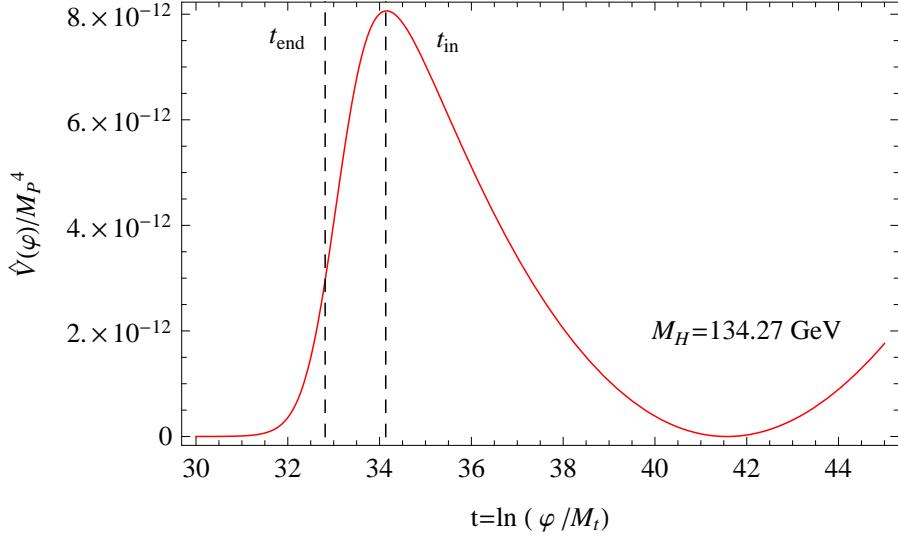


Fig. 5. The effective potential for the instability threshold  $M_H^{\text{inst}} = 134.27$  GeV. A false vacuum occurs at the instability scale  $t_{\text{inst}} \simeq 41.6$ ,  $\varphi \sim 80M_P$ . An inflationary domain for a  $N = 60$  CMB perturbation (ruled out by the WMAP bounds) is marked by dashed lines.

the non-perturbative domain of large over-Planckian scales due to a fast drop of  $\hat{V} \sim \lambda/\xi^2$  to zero. This, in turn, follows from the fact that  $\xi(t)$  grows much faster than  $\lambda(t)$  when they both start approaching their Landau pole.<sup>15)</sup>

The location  $\varphi_0$  of the probability peak and its quantum width can be found in analytical form, and their derivation shows the crucial role of the running  $\mathbf{A}_I(t)$  for the formation of initial conditions for inflation. Indeed, the exponential of (6.1) for  $M_P^2/\xi\varphi^2 \ll 1$  in view of the RG equations for  $\lambda$  and  $\xi$  with beta functions (3.6)–(3.7) has an extremum satisfying the equation

$$\varphi \frac{d\Gamma}{d\varphi} = \frac{d\Gamma}{dt} = -\frac{6\xi^2}{\lambda} \left( \mathbf{A}_I + \frac{64\pi^2 M_P^2}{\xi Z^2 \varphi^2} \right) = 0, \quad (6.2)$$

where we neglect higher order terms in  $M_P^2/\xi Z^2 \varphi^2$  and  $\mathbf{A}_I/64\pi^2$  (extending beyond the one-loop order). Here,  $\mathbf{A}_I$  is the anomalous scaling (2.2) and (2.5) which is negative in the domain of interest and, thus, guarantees the existence of the solution for the probability peak,

$$\varphi_0^2 = -\frac{64\pi^2 M_P^2}{\xi \mathbf{A}_I Z^2} \Big|_{t=t_0}. \quad (6.3)$$

In the CMB-compatible range of  $M_H$  its running starts from the range  $-36 \lesssim \mathbf{A}_I(0) \lesssim -23$  at the EW scale and reaches small but still negative values in the range  $-11 \lesssim \mathbf{A}_I(t_{\text{end}}) \lesssim -2$  at the inflation scale. Also, the running of  $\mathbf{A}_I(t)$  and  $Z(t)$  is very slow – the quantities belonging to the two-loop order – and the duration of inflation is very short  $t_0 \sim t_{\text{in}} \simeq t_{\text{end}} + 2$ . Therefore,  $\mathbf{A}_I(t_0) \simeq \mathbf{A}_I(t_{\text{end}})$ , and these

estimates apply also to  $\mathbf{A}_I(t_0)$ . As a result, the second derivative of the tunneling on-shell action is positive and very large,  $d^2\Gamma_-/dt^2 \simeq -(12\xi^2/\lambda)\mathbf{A}_I \gg 1$ , which gives an extremely small value of the quantum width of the probability peak,

$$\frac{\Delta\varphi^2}{\varphi_0^2} = -\frac{\lambda}{12\xi^2}\frac{1}{\mathbf{A}_I}\bigg|_{t=t_0} \sim 10^{-10}. \quad (6.4)$$

This width is about  $(24\pi^2/|\mathbf{A}_I|)^{1/2}$  times – one order of magnitude – higher than the CMB perturbation at the pivotal wavelength  $k^{-1} = 500$  Mpc (which we choose to correspond to  $N = 60$ ). The point  $\varphi_{\text{in}}$  of the horizon crossing of this perturbation (and other CMB waves with different  $N$ 's) easily follows from equation (2.6) which in view of  $\mathbf{A}_I(t_0) \simeq \mathbf{A}_I(t_{\text{end}})$  takes the form

$$\frac{\varphi_{\text{in}}^2}{\varphi_0^2} = 1 - \exp\left(-N\frac{|\mathbf{A}_I(t_{\text{end}})|}{48\pi^2}\right). \quad (6.5)$$

It indicates that for wavelengths longer than the pivotal one the instant of horizon crossing approaches the moment of “creation” of the Universe, but always stays chronologically succeeding it, as it must.

## §7. Conclusions and discussion

We have found that the considered model looks remarkably consistent with CMB observations in the Higgs mass range

$$135.6 \text{ GeV} \lesssim M_H \lesssim 184.5 \text{ GeV}, \quad (7.1)$$

which is very close to the widely accepted range dictated by electroweak vacuum stability and perturbation theory bounds.

This result is based on the observation that for large  $\xi \gg 1$  the effect of SM phenomenology on inflation is universally encoded in one quantity – the anomalous scaling  $\mathbf{A}_I$ . It was earlier suggested in<sup>1)</sup> for a generic gauge theory, and in SM it is dominated by contributions of heavy particles – ( $W^\pm$ ,  $Z$ )-bosons, top quark and Goldstone modes. This quantity is forced to run in view of RG resummation of leading logarithms, and this running raises a large negative EW value of  $\mathbf{A}_I$  to a *small negative* value at the inflation scale. Ultimately this leads to the admissible range of Higgs masses very close to the conventional SM range.

Qualitatively these conclusions are close to those of<sup>5)</sup> and<sup>6)</sup> though our bounds on the SM Higgs mass are much more sensitive to the CMB data. Therefore, the latter can be considered as a test of the SM theory complimentary to LHC and other collider experiments. The source of this difference from<sup>5),6)</sup> can be ascribed to the gauge and parametrization (conformal frame) dependence of the off-shell effective action (along with the omission of Goldstone modes contribution in<sup>5)</sup>) – an issue which is discussed in much detail in<sup>15)</sup> and which is expected to be resolved in future publications.

We have also shown the naturalness of the gradient and curvature expansion in this model, which is guaranteed within the conventional perturbation theory range of

SM,  $\lambda/16\pi^2 \ll 1$ , and holds in the whole range of the CMB compatible Higgs mass (7.1). This result is achieved by the background field resummation of weak field perturbation theory leading to the replacement of the fundamental Planck mass in the known cutoff  $4\pi M_P/\xi^{28)}$ ,<sup>29)</sup> by the effective one. Partly (modulo corrections to inflaton potential, which are unlikely to spoil its shape) this refutes objections of<sup>28),29)</sup> based on the analysis of scattering amplitudes in EW vacuum background. Smallness of the cutoff in this background does not contradict physical bounds on the Higgs mass originating from CMB data for the following reasons. Determination of  $M_H$  of course takes place at the TeV scale much below the non-minimal Higgs cutoff  $4\pi M_P/\xi$ , whereas inflationary dynamics and CMB formation occur for  $\lambda/16\pi^2 \ll 1$  below the *running* cutoff  $\Lambda(\varphi) = 4\pi\varphi/\sqrt{\xi}$ . It is the phenomenon of inflation which due to exponentially large stretching brings these two scales in touch and allows us to probe the physics of underlying SM by CMB observations at the 500 Mpc wavelength scale.

We also applied the quantum cosmology concept to derive initial conditions in this particular model of inflation. Specifically, we used the recently suggested path integral formulation of the tunneling cosmological state<sup>18)</sup>, which admits a consistent renormalization scheme and becomes indispensable in the case when quantum effects play a dominant role. In this way a complete cosmological scenario was obtained, embracing the formation of initial conditions for the inflationary background (in the form of a sharp probability peak in the inflaton field distribution) and the ongoing generation of the CMB perturbations on this background. Interestingly, the behavior of the running anomalous scaling  $\mathbf{A}_I(t) < 0$ , which is crucially important for the CMB formation and the corresponding Higgs mass bounds, also guarantees the existence of the obtained probability peak.<sup>18)</sup> The quantum width of this peak is one order of magnitude higher than the amplitude of the CMB spectrum at the pivotal wavelength, which could entail interesting observational consequences. Unfortunately, this quantum width is hardly measurable directly because it corresponds to an infinite wavelength perturbation (a formal limit of  $N \rightarrow \infty$  in (6.5)), but indirect effects of this quantum trembling of the cosmological background deserve further study.

To summarize, the obtained results bring to life a convincing unification of quantum cosmology with the particle phenomenology of the SM, inflation theory, and CMB observations. They support the hypothesis that an appropriately extended Standard Model<sup>43)</sup> can be a consistent quantum field theory all the way up to quantum gravity and perhaps explain the fundamentals of all major phenomena in early and late cosmology.

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